Calculation of Higher Order Effects in Electron-Positron Pair Production in Relativistic Heavy Ion Collisions

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Abstract

We present a calculation of higher order effects for the impact parameter dependent probability for single and multiple electron-positron pairs in (peripheral) relativistic heavy ion collisions. Also total cross sections are given for SPS and RHIC energies. We make use of the expression derived recently by several groups where the summation of all higher orders can be done analytically in the high energy limit. An astonishing result is that the cross section, that is, integrating over all impact parameters, is found to be identical to the lowest-order Born result for symmetric collisions. For the probability itself on the other hand we find rather large effects at small impact parameters compared to the lowest order results, which translate to large effects for the cross section for multiple pair production.

12.20.-m;34.50.-s;25.75.-q

Electron-Positron pair production in (peripheral) relativistic heavy ion collisions has attracted some interest recently due to the observation that at the relativistic heavy ion collider RHIC and LHC, the probability for this process calculated in perturbation theory violates unitarity, that is, gets larger than one, even for impact parameters of the order of the Compton wave length $\lambda_C \approx 386$ fm. This fact was first shown in [1,2]. The unitarity violation was then studied in a number of articles, taking into account higher order processes in [2–6]. It was found that the inclusion of these higher-order processes leads to the restoration of unitarity but also to new effects, mainly the production of multiple pairs. All studies also found that the probability for N-pair production can be approximately described by a Poisson distribution. Therefore the probability from perturbation theory (in the following called "reduced probability") has to be interpreted as the average number of pairs produced in one collision. Deviations from the Poisson distribution where studied for small impact parameters in [5] and found to be rather small.

Calculations of the impact parameter dependent probability in the external field approximation where calculated exactly in lowest order for small impact parameters b [7] and later also for all impact parameters [8,9]. The total cross section for pair production (that is, integrated over impact parameters) was also calculated. Using a Monte Carlo approach it was studied in [10]. An analytical form of the differential cross section was found in [6]. Cross sections for multiple pair production were also given there.

One open question is still the importance of higher order corrections — or even nonperturbative effects — coming from the large value of the effective coupling constant $Z\alpha \approx 0.6$. Coupled channel calculations have been done at smaller energies (up to $\gamma \approx 2$ in the collider frame). They always predicted a much larger probability compared to perturbation theory [11,12]. But recently the accuracy of these calculations has been questioned and by using a larger basis set smaller results were found. In the end results were only a factor of four larger than the perturbative results [13], which is also in agreement with calculations using a spline-approach [14].

In a recent article the summation of the effect of the target to all orders was studied in the high energy limit (that is up to lowest order in $1/\gamma$) for the related problem of bound-free pair production [15]. Here the electron is created into a bound state of one of the ions. It was found that the summation to all orders could be done analytically and a fairly simple expression was found. The calculated probability for bound-free pair production was found to be slightly smaller than the first order calculations.

A number of authors have in the mean time extended this approach also to the calculation of (free) electron-positron pairs. In a first article [16] it was shown that here the summation can be done analytically again, leading to a rather simple modification of the matrix element. The authors of [17] come to the same conclusion; they show also by integrating over the impact parameter that the cross section becomes identical to the lowest order Born result. In [18] the same conclusion is given. In [19] the scattering of electrons in the field of colliding nuclei is studied, a problem which is closely related to pair-production.

Of course there remain a few questions that still need to be addressed. The calculations were done using the Dirac-sea picture (that is, starting with an electron with negative energy in the initial state) and are therefore one-particle approximations of the full problem. The applicability of this approximation for situations where strong fields produce many particles has still to be investigated. The use of the reduced probability in the Poisson distribution to

get multiple pair production was only derived in the (usual) Feynman picture. Also only a part of all possible diagrams are included in the approach. Fig. 1(A) shows typical diagrams which are included, whereas those of Fig. 1(B) are not. In [19] it is argued that this second class of diagrams vanish in the limit $\gamma \to \infty$ for electron scattering, therefore the neglect of them seems to be justified. But such a rigorous proof is still needed for pair production. Finally it is well known from calculations with the approach of Bethe, Maximom and Davis that Coulomb corrections exist and are not small at the energies considered [1,20]. This seems to be in contrast to the observation that the cross section should be identical to the lowest-order one for ion collisions. The most likely explanation of their absence is, that they are of subleading order in $1/\gamma$ and were therefore dropped.

It is our main aim to present a numerical calculation of the higher order effects in these expressions given in these articles in order to study their magnitude and their practical implications. Whereas the cross section for one-pair production is dominated by the large impact parameters, multiple-pair production is more sensitive to small impact parameters. Therefore we try to see whether a measurement of multiple pairs can be used to look for these higher order effects.

We make use of the expression Eq. (54) of [17]. In addition we also keep all effects of finite γ in the expression. Written in our notation the (reduced) differential probability is

$$P(p_+, p_-, b) = \int d^2\Delta q \tilde{P}(p_+, p_-, \Delta q) \exp(i\Delta\vec{q}\vec{b}) \tag{1}$$

where p_+ and p_- are the momenta of positron and electron and the Fourier-transform of the probability $\tilde{P}$ is given for symmetric collisions ($Z = Z_A = Z_B$) by:

$$\begin{aligned}
&\tilde{P}(p_+, p_-, \Delta q) = \frac{4\eta^4}{\beta^2} \int d^2q \\
&\Big\{[-q^2]^{1+i\eta}[-q'^2]^{1-i\eta}[-(q-(p_+ + p_-)^2)]^{1+i\eta} \\
&[-(q'-(p_+ + p_-)^2)]^{1-i\eta}\Big\}^{-1} \mathrm{Tr}\Big\{(\not{p}_- + m) \\
&\left[\frac{\not{w}_1(\not{p}_- - \not{q} + m)\not{w}_2}{-(q-p_-)^2 + m^2} + \frac{\not{w}_2(\not{q} - \not{p}_+ + m)\not{w}_1}{-(q-p_+)^2 + m^2}\right](\not{p}_+ - m) \\
&\left[\frac{\not{w}_2(\not{p}_- - \not{q}' + m)\not{w}_1}{-(q'-p_-)^2 + m^2} + \frac{\not{w}_1(\not{q}' - \not{p}_+ + m)\not{w}_2}{-(q'-p_+)^2 + m^2}\right]\Big\},
\end{aligned} \tag{2}$$

with $\eta = Z\alpha$, $q' = q + \Delta q$, $w_1 = (1, 0, 0, \beta)$, $w_2 = (1, 0, 0, -\beta)$ and m the electron mass. This expression is almost identical to the one from perturbation theory, as given in [9]; the only difference is the additional exponents $1 \pm i\eta$ for the photon propagator. This fact was already observed in [17,18]. Integrating over b leads to a delta-function $\delta(\Delta q)$. Then the photon propagators are just complex conjugate to each other and only the absolute value enters. Therefore the cross section calculations of [10,6] are exact in the high energy limit.

Here we want to study the effect of the higher orders for small impact parameters. The total (single pair) cross section is completely dominated by the large impact parameters, especially in the high energy limit. Stronger deviations are to be expected mainly for small b, especially if b gets smaller than the Compton wavelength $\lambda_C \approx 386 fm$.

The new expression has a complex exponent, which makes the expressions oscillatory. Therefore a direct Monte Carlo integration is not possible. We rewrite it into a form with only standard Feynman integrals. We start by applying the usual Feynman-trick to group a product of two denominator into a single one, integrating over an auxiliary parameter. This trick is normally used for integer exponents. But it is easy to see by looking at a derivation of this in terms of B-functions (see, e.g., [21]), that the same expression also hold for complex exponents. We use it here in the following form:

$$\frac{1}{C^{1+i\eta}D^{1-i\eta}} = \frac{1}{B(1+i\eta,1-i\eta)} \int_0^1 \frac{w^{1+i\eta}(1-w)^{1-i\eta}dw}{[wC+(1-w)D]^2} \tag{3}$$

Rewriting both photon propagators, we get two auxiliary integrations: w_A and w_B. In the new denominator the factor $\pm i\eta$ just cancel and the remaining expression is of the form of (the square of) a propagator. The remaining two-dimensional integral over q can then be done in the same way as discussed in [7,9,6]. We integrate over all final states of the electron and positron using VEGAS [22].

Due to the oscillatory behavior of the numerator at the boundaries, a direct numerical evaluation of the auxiliary integrals is not useful. We therefore expand $\tilde{P}$ for each Δq in terms of polynomials of the form $[w_A(1-w_A)]^k[w_B(1-w_B)]^l$. Terms up to $k,l=5$ have been included in a fit, but convergence is already found with smaller exponents. The integrals over these polynomials can be expressed now in terms of the B-functions. Our approach has the advantage that the coefficients of the polynomial expansion are independent of η, apart from the trivial η^4 dependence. Therefore results for arbitrary η can be calculated with no extra effort. Fourier-transforming this expression now with respect to Δq gives us $P(b)$. The total cross section, that is, integrated over b, is given directly by $\sigma = (2\pi)^2\tilde{P}(\Delta q = 0)$.

We have calculated $\tilde{P}(\Delta q)$ and $P(b)$ for both SPS (Pb-Pb collisions at $\gamma = 10$) and RHIC (Au-Au collisions at $\gamma = 100$) conditions. As a check of the correctness of our calculations, we can compare them for $\eta \to 0$ with the perturbative results in [9], where a different approach was used. We get perfect agreement between those two, giving us confidence in our procedure.

Figure 2 shows the Fourier transform for $\gamma = 100$ and for different η. The effect of the higher orders is quite large, making $\tilde{P}$ smaller up to about 30%. The shape of the curve itself is changed only slightly. For $\Delta q \to 0$ all curves coincide with each other. This has to be the case as the total cross section is identical to the lowest order one.

Figure 3 shows the impact parameter dependent cross section $d\sigma/db = 2\pi b P(b)$ for different values of η. A deviation is only seen for small impact parameters, where it is quite large.

Making now use of the Poisson distribution we can calculate probabilities for multiple pair production. For $\gamma = 100$ we get the results shown in Fig. 4. The higher order processes reduce the multiple pair production probabilities, but the probability for two-pair production is still large. Integrating over b we get the total cross sections as given in table I. For the single-pair cross section we use the approach of [6][1]. It is clearly seen that the cross section

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[1]This could be improved by taking into account the Poisson distribution and also a lower bound

for multiple pair production is sensitive to the higher order effects in both cases. Therefore measuring them seems to be a practical way to look for these effects.

In conclusion, we have calculated total probabilities for one and multiple pair production including higher order effects using the expression of [17]. This approach effectively sums up all higher order diagrams in the high energy limit. We have calculated cross sections for multiple pair production and have found both probabilities and cross sections to be substantially smaller for the full calculation compared to the perturbative one. Therefore a measurement of this cross section should allow to really see these higher order QED effects in an experiment.

Higher order effects can only be seen if one uses observables which are sensitive to small impact parameters. Multiple pair production is one such possibility. Depending on the experimental situation one could also think of other ways, e.g., the production of other particles together with an e^+-e^- pair.

In this article we have concentrated on "global properties" of the pair production, that is, total probabilities and cross sections. Our main aim was to demonstrate that calculations are possible. In order to see whether experiments will be able to see these effects, more differential studies are needed. We will present these and also details of the calculations in an upcoming publication.

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for the impact parameters. As these effects are rather small, we have neglected them here.

REFERENCES

[1] C.A. Bertulani and G. Baur, Phys. Rep. **163**, 299 (1988).
[2] G. Baur, Phys. Rev. A**42**, 5736 (1990).
[3] M.J. Rhoades-Brown and J. Weneser, Phys. Rev. A**44**, 330 (1991).
[4] C. Best, W. Greiner, and G. Soff, Phys. Rev. A**46**, 261 (1992).
[5] K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A**51**, 998 (1995).
[6] A. Alscher, K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A**55**, 396 (1997).
[7] K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A**49**, 1584 (1994).
[8] M. C. Güçlü, J. C. Wells, A. S. Umar, M. R. Strayer, and D. J. Ernst, Phys. Rev. A**51**, 1836 (1995)
[9] K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A**51**, 1874 (1995).
[10] C. Bottcher and M.R. Strayer, Phys. Rev. D**39**, 1330 (1989).
[11] K. Momberger, G. Soff, and W. Scheid, Z. Phys. D**18**, 133 (1991).
[12] K. Rumrich, G. Soff, and W. Greiner, Phys. Rev. A**47**, 215 (1993).
[13] K. Momberger, A. Belkacem, and A. H. Sørensen, Europhys. Lett. **32**, 401 (1995).
[14] J. Thiel, J. Hoffstadt, N. Grün, and W. Scheid, Z. Phys. D**34**, 21 (1995)
J. Thiel, thesis Universität Giessen (1993), unpublished.
[15] A. J. Baltz, Phys. Rev. Lett. **78**, 1231 (1997).
[16] B. Segev and J. C. Wells,Phys. Rev. A**57**, 1849 (1998).
[17] A. J. Baltz and L. McLerran, *Two Center Light Cone Calculation of Pair Production Induced by Ultrarelativistic Heavy Ions*, nucl-th/9804042.
[18] B. Segev and J. C. Wells, physics/9805013 (1998).
[19] U. Eichmann, J. Reinhardt, S. Schramm, and W. Greiner, *Electron Propagation in the Field of Colliding Nuclei at Ultrarelativistic Energies*, nucl-th/9804064.
U. Eichmann, J. Reinhardt,and W. Greiner, *Coulomb Effects on Electromagnetic Pair Production in Ultrarelativistic Heavy-Ion Collisions*, nucl-th/9806031.
[20] D. Ivanov, E. A. Kuraev, A. Schiller, and V. G. Serbo, hep-ph/9807311.
[21] M. E. Peskin and D. V. Schroeder, “An Introduction to Quantum Field Theory”, Addison-Wesley, 1995.
[22] G.P. Lepage, J. Comp. Phys. **27**, 192 (1978).

FIGURES

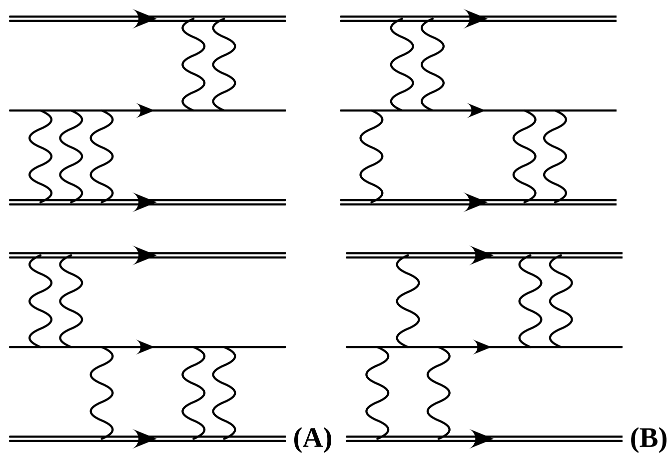

FIG. 1. Diagrams of the type (A) are included in the matrix element. Diagrams of type (B) are assumed to be subdominant for large γ.

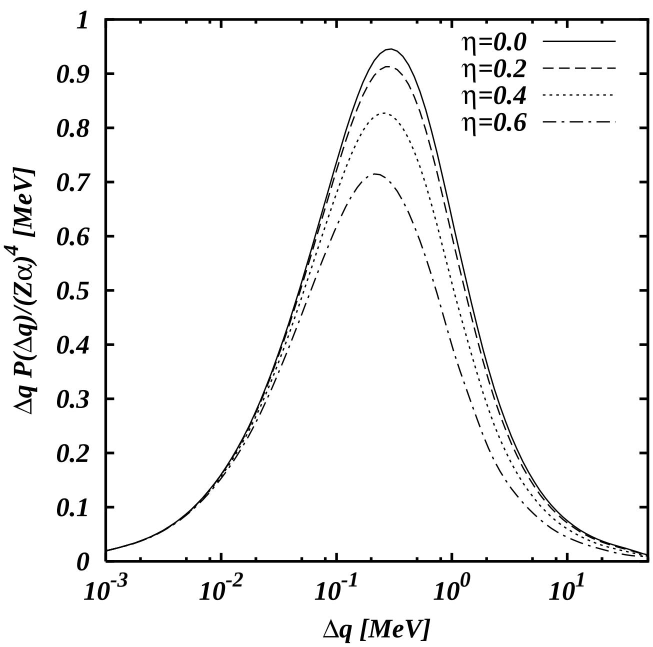

FIG. 2. $\Delta q P(\Delta q)$, the Fourier-transform of the total probability is shown for $\gamma = 100$. The trivial $\eta^4 = (Z\alpha)^4$ dependence is divided out. Shown are the results for different η for the full calculation.

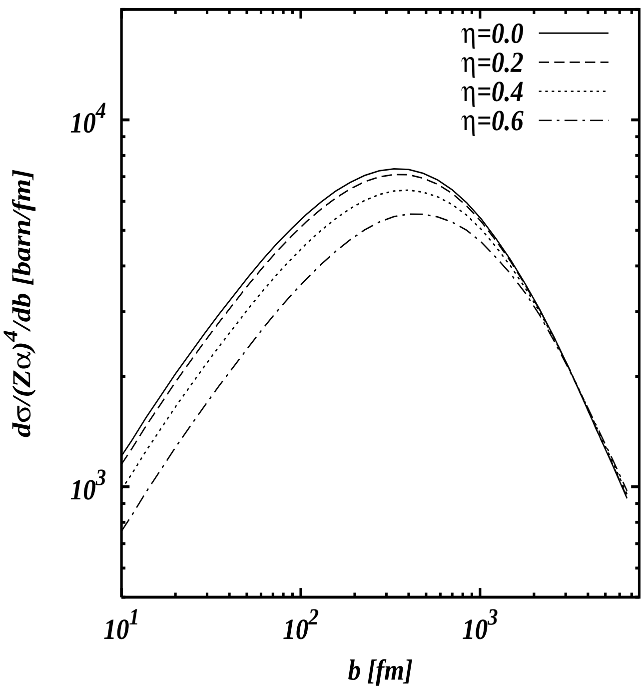

FIG. 3. Shown is the differential cross section $d\sigma/db$ for $\gamma = 100$ and for different values of η. The trivial $\eta^4 = (Z\alpha)^4$ dependence is divided out. At small impact parameters the cross section is reduced quite substantially.

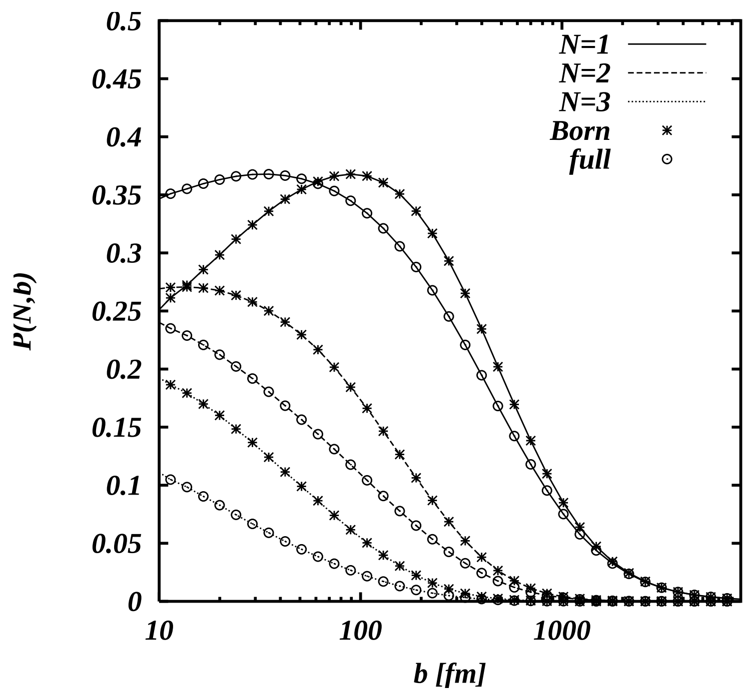

FIG. 4. The impact parameter dependent probability to produce N pairs is shown for up to three pairs. Shown are results for the lowest order Born result (stars) and also the full calculation (circles) both for $\gamma = 100$ and Au-Au collisions.

TABLES

N	Born (barn)	full (barn)
$\gamma = 10$, Pb-Pb ($Z = 82$,$\eta = 0.59$)		
1	4.21k	4.21k
1	123	84.4
2	8.61	3.88
3	0.713	0.212
$\gamma = 100$, Au-Au ($Z = 79$,$\eta = 0.57$)		
1	34k	34k
2	893	624
3	113	53.9
4	18.9	6.04

TABLE I. *Total cross section for one and multiple pair production are given. Shown are the results for SPS/CERN and RHIC conditions. For the results for $N = 1$ the approach of [6] was used.*